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A corrected incompressible SPH method for fixed body wave impact simulation

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Abstract

The smoothed particle hydrodynamics method (SPH) are emerging as potential tools for studying water wave related problems, in particular those with a rapidly moving free surface. And the incompressible smoothed particle hydrodynamics (ISPH) method has been shown to be accurate and stable for many problems than the traditional SPH by many papers in literature. In this study, the cases about wave breaking and solitary wave are simulated. Their results are compared with available analytical, experimental, and other numerical data found in literatures and reasonably good agreement is achieved. And the corrected incompressible smoothed particle hydrodynamics has been shown to be useful for simulating fixed body wave impact.

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Keywords: Incompressible SPH; correction; wave impact; wave breaking; solitary wave;

1. Introduction

As a purely Lagrangian method, the Smoothed Particle Hydrodynamics (SPH) has been successfully applied in a large number of free surface flows simulations [1, 2]. The incompressibility of fluid has two ways. The first one is weakly compressible SPH (WCSPH), The second formulation is incompressible SPH, also called ISPH.

The application of boundary conditions is problematic in the SPH technique. Different researcher may use different handling methods, like Shao and Lo Edmond [3] and Lee et al. [4]. ISPH method should identify the free surface particle. Very recently, an effective scheme was proposed by Ma and Zhou [5] for using the MLPG_R method to model breaking waves. Additionally, distribution of particles always becomes disorderly when modelling

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violent waves, even they are regularly distributed at the start of simulation. The formation of ill particle distributions during the simulation may result in error.

This paper suggests a corrected ISPH algorithm for the study of wave fixed structure interactions. In this method the improved solid boundary handling method following Lo and Shao [6] and the new free surface identification scheme following Ma and Zhou [5] will be applied into the model. And an over mirroring problem might appear in boundary corners also gets the special treatments. The artificial particle displacement procedure has been prescribed to prevent the particle clustering. In the final model applications, two different fixed body wave impact cases are presented to test the model capability and versatility.

2. SPH methodology

2.1. SPH solution algorithms

In SPH method, The Lagrangian form of the Navier-Stokes equation is written as follows

$$\frac{\mathrm{D}\rho}{\mathrm{D}t}+\rho\nabla\cdot\boldsymbol{u}=0 \tag{1}$$

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t}=-\frac{1}{\rho}\nabla p+\boldsymbol{g}+\nu\nabla^2\boldsymbol{u} \tag{2}$$

Incompressibility is enforced in a correction step of the time integration by setting $D\rho/Dt=0$ at each particle. So in incompressible SPH, Eq.(1) can be changed to

$$\nabla\cdot\boldsymbol{u}=0 \tag{3}$$

The prediction–correction scheme of the I-SPH method consists of two steps. In the prediction step, an intermediate particle velocity and position are obtained by

$$\boldsymbol{u}_*=\boldsymbol{u}_t+\Delta\boldsymbol{u}_* \tag{4}$$

$$\Delta\boldsymbol{u}_*=(\boldsymbol{g}+\nu\nabla^2\boldsymbol{u})\Delta t \tag{5}$$

$$\boldsymbol{r}_*=\boldsymbol{r}_t+\boldsymbol{u}_*\Delta t \tag{6}$$

The velocity change during the correction step is

$$\Delta\boldsymbol{u}_{**}=-\frac{1}{\rho}\nabla P_{t+1}\Delta t \tag{7}$$

Velocity and position on $t+1$ step is $\boldsymbol{u}_{t+1}$, $\boldsymbol{r}_{t+1}$, which can be defined as

$$\mathbf{u}_{t+1}=\mathbf{u}_*+\Delta\mathbf{u}_{**} \tag{8}$$

$$\mathbf{r}_{t+1}=\mathbf{r}_t+\frac{\mathbf{u}_t+\mathbf{u}_{t+1}}{2}\Delta t \tag{9}$$

Combined Eqs.(3) and (7), it can get the pressure Poisson equation, which can be shown as

$$\nabla^2 P_{t+1}=\frac{\rho\nabla\cdot\mathbf{u}_*}{\Delta t} \tag{10}$$

Hu and Adams [7] using the combined method of divergence-free and density- invariance, it can be shown as

$$\nabla^2 P_{t+1}=\alpha\frac{\rho-\rho^*}{\Delta t^2}+(\alpha-1)\frac{\rho\nabla\cdot\mathbf{u}_*}{\Delta t} \tag{11}$$

It is applied in this paper, and $\alpha=0.01$ for all numerical results.

2.2. Instabilities and their possible remedies in the SPH method

To prevent the particle clustering, the trajectory of particles can be disturbed by adding relatively small artificial displacement δr_i^k to the advection of particles, which is similar to Xu and Stansby [8]. δr_i^k is defined as

$$\delta r_i^k=\beta\sum_j^N\frac{r_{ij}^k}{r_{ij}^3}r_o^2\nu_{\max}\Delta t \tag{12}$$

where δr_i^k is an artificial particle displacement vector. β is a problem -dependent parameter and kept constant and equal to 0.01~0.1. Here, the cut-off distance can be approximated as $r_0 = \sum_{j=1}^{N} r_{ij} / N$, N is the number of neighbors for particle i in its support domain. v_{max} is the largest particle velocity in the system.

3. Wall boundary and free surface

Wall boundaries are also simulated by particles, which is similar to the method of Lo and Shao [6]. The wall has fixed particles located at the wall itself. Then, for the interaction of inner fluid particle a with particle b , an additional interaction between a and b mirror b_{mir} is included if a and b are close to the wall. The mirror particle b_{mir} is shown in Fig. 1(a). Additionally, some special treatment should be taken in boundary corners as shown in Fig. 1(b), in which when an inner fluid particle searches for its neighbors in the over-mirror region, only the mirrors across the same boundary are allowed to be captured.

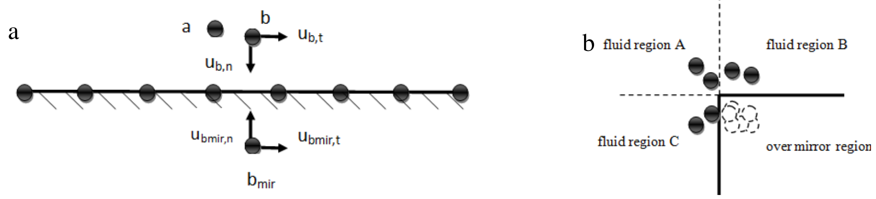

Fig.1. Wall boundary treatment (a) relationship between inner, mirror, and wall particles.(b) over mirror treatment

For violent breakings, the particles on the free surface can become inner particles and inner particles can become free surface particles. Therefore, free surface particles have to be identified at every time step after breaking occurs. Very recently, another scheme was proposed by Ma and Zhou [5] for using the MLPG_R method to model breaking waves. This efficient method is applied in this paper.

4. Numerical tests

4.1 Dam breaking

The dam breaking is a widely used test case for impulsively started, rapidly evolving free-surface flows. In this section, dam breaking with vertical wall, 45° slopes and 135° slopes are simulated , the model of dam breaking is given by Fig.2, $a = 0.5m$, $H / a = 2.0$, $L/a = 4.0$, all variables and parameters are non-dimensionalzed using a and g.

In order to verification of the accuracy results obtained by ISPH simulating dam breaking with the vertical wall, it gives the comparison with experimental results from Matin and Moyce [9], which includes the wave front shown in Fig.3(a) and column height in Fig.3(b), the numerical results can get a good agreement with experimental results.

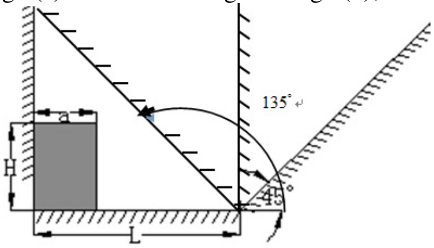

Fig.2. Sketch of dam breaking model

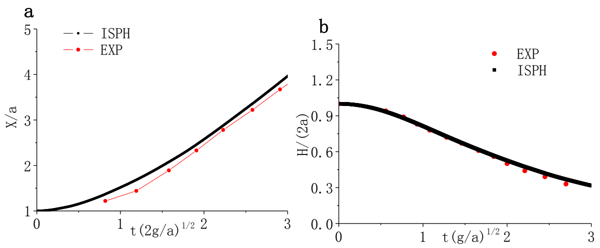

Fig.3. (a) Time history of wave front (b) Time history of column height on left wall

The snapshot of pressure contour of at different time can be shown in Fig.4, The case includes the phenomena of running up, over-turning and re-entering. According to the snapshot of pressure contour of three cases, the pressure distribution is regular and smooth.

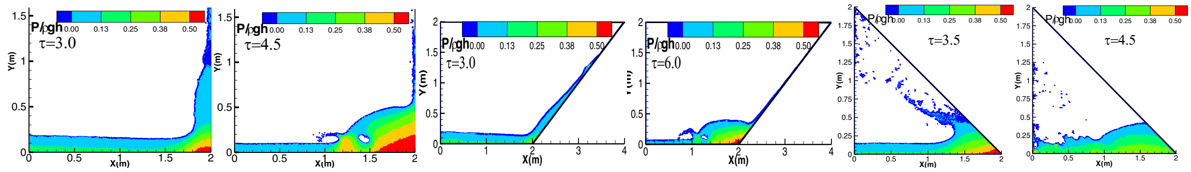

Fig.4. Pressure distribution of different time with different wall boundary

4.2 Solitary wave propagation

To test the efficiency of ISPH scheme in addressing complex deformation of free surfaces and non-linear flow interactions with the fixed structure, a solitary wave propagation is simulated. In this section, all variables and parameters are non-dimensionalzed using d and g.

The analytical solution for the wave profile can be derived from the Boussinesq equation [10].

$$\eta(x,t) = a\,\mathrm{sec}h^2\left[\sqrt{\frac{3a}{4d^3}}(x-ct)\right] \tag{13}$$

where η is water surface elevation, a is wave amplitude, d is water depth and $c=\sqrt{g(d+a)}$ is the solitary wave celerity. The motion of the wavemaker is given by Ma and Zhou [5]:

$$x_p(\tau) = \sqrt{\frac{4a}{3d}}\,d\left[\tanh\left\{\sqrt{3a/4d^3}\left[c\tau - x(\tau) - \lambda\right]\right\} + \tanh(\lambda\sqrt{3a/4d^3})\right] \tag{14}$$

The schematic diagram of the problem domain is shown in Fig. 5(a). The wave height $a = 0.24m$.

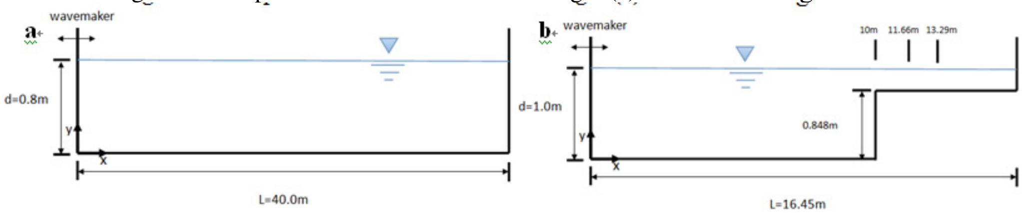

Fig. 5. Sketch of solitary wave propagation model (a) in a constant water depth (b) over a step

The normalized pressure field under the solitary wave crest at $\tau = 22.5$ is shown in Fig. 6. Figure 7 shows the comparison between the analytical and the simulated wave profile. It can be seen that the numerical wave profile agrees well with the analytical one.

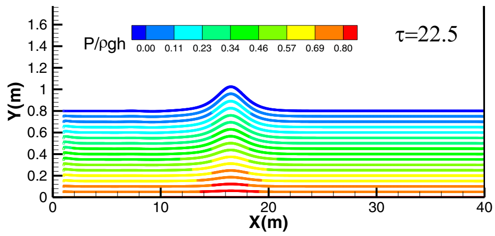

Fig.6. Normalized pressure field

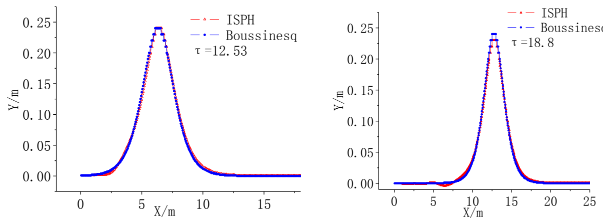

Fig.7. Comparison for the time histories for the free surface elevations

To further validate the corrected ISPH model, the solitary wave generated by a wave maker at one end and then propagating it over a step is considered. This case has been experimentally studied by Yasuda, Mutsuda and Mizutani [11]. The same problem was also investigated by Ma and Zhou [5], using the MLPG method and Chowdhury and Sannasiraj [12], using the traditional SPH method. Sketch of the problem domain is shown in Fig. 5.(b), in which there are three wave probes P1, P2 and P3 located at x= 10.0, 11.66 and 13.29.

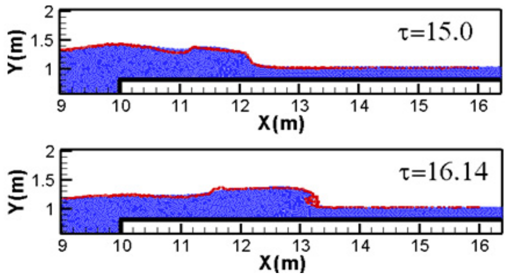

Fig.8. Comparison for wave profile at different steps: blue -ISPH, red dots- MLPG

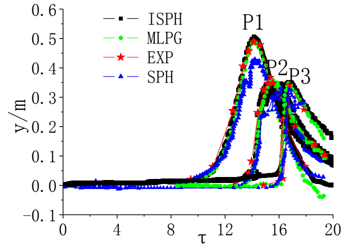

Fig.9. Comparison of wave elevation at different probes between experimental data, ISPH, MLPG, and SPH

Figure 8 shows a direct comparison between the corrected ISPH model and the MLPG model for the free surface profile at two different instants, the results agree well with each other. The comparisons for the wave elevation at the probes are given in Fig.9. It can be found from these figures that the corrected ISPH model predict well with method MLPG and the experimental data.

5. Conclusion

In this paper, a corrected incompressible SPH method is presented to simulate fixed body wave impact. The method employs an efficient solid boundary handling method and free surface particle identification method, which can get stable and reliable results for complex boundary condition and free surface simulation. A correction has been also prescribed for the particle position to prevent the particle clustering. The numerical tests show that the corrected ISPH method can yield satisfactory results for fixed body wave impact simulation.

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